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Experimental study of human-induced dynamic forces due to jumping on a perceptibly moving structure

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Abstract

This paper describes the first ever direct measurements of human-induced dynamic forces due to jumping on a force platform which is moving. Therefore, the paper addresses the increasingly important issue of jumping on a flexible structure that can move perceptibly. A unique test rig, developed to permit a person to jump on an idealised single-degree-of-freedom system with variable natural frequency and mass, is described and the test methodology explained. A set of representative results, for different rig natural frequencies (2–6 and 16 Hz) and a range of achievable jumping frequencies (1–3.5 Hz), is presented. This clearly demonstrates the effect of the flexibility of the structure on the levels of dynamic response and force that can potentially be generated by humans when they feel the motion of the structure on which they jump. The acceleration and displacement responses show significant peaks when the jumping frequency is in the region of half the natural frequency and of the natural frequency itself. This indicates that the first and second harmonic of the human-induced forcing functions are exciting resonant response, as would be expected. However, it is also shown that, for the test rig configuration chosen, it was not physically possible to jump at frequencies close to the natural frequency when the structural motion was significant. This is a new finding thought to be due to the limitations imposed by the projectile motion of the human test subject. It is also apparent that the contact ratio (ratio of time in contact with the platform/period of jumping), determined from the measured jumping force time history, increases in the regions of peak response and does not ever fall below a value of 0.5. This is a considerably higher contact ratio value than was established in the past in similar jumping tests performed on stiff and not perceptibly moving surfaces. As a consequence of the variation of contact ratio, the amplitudes of the force harmonics do not vary only with the jumping frequency, which is a widely known fact, but also with the ratio of jumping to natural frequency, which is linked to the amount of motion of the test rig.

1. Introduction

There is currently a considerable interest in the UK in the effect that people can have when moving upon flexible structures whose motion can be perceived [1]. In particular, this issue has emerged in recent years due

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to the changing nature of the design of sports stadia, where long span and cantilevered seating decks are becoming more popular because of the improved sight lines offered to spectators. There has also been a tendency to view stadia as multipurpose venues, with the possibility of gaining additional income from pop concerts.

The effect of these changes is that seating decks of modern stadia tend to have low vertical natural frequencies, often in the 2–4 Hz range. Such low frequencies are in the range that may be excited by crowds undergoing sudden or rhythmic motion, the most severe being bouncing and jumping. The former is also known as 'jouncing' or 'bobbing', and it happens when the person is in continuous contact with the structure. Jumping occurs when a person moves his/her body up and down and is airborne for some of the time. These motions can occur most strongly around 2–3 Hz for jumping whereas bouncing is physically possible for frequencies as high as 4.5 or even 5.0 Hz. It is well known that the 'near periodic' but discontinuous force time history generated when a subject jumps may be represented reasonably well by a truncated series of harmonics. Therefore, jumping at 2 Hz will cause harmonic excitation components at integer multiples of 2 Hz (i.e. at 2, 4, 6 Hz, etc.). Thus, the potential exists for a seating deck structure to respond considerably if one or more of these jumping frequency harmonics matches one or more of its natural frequencies and creates resonance. In the UK, design guidelines have been laid down for such problems. For example, the Guide to Safety at Sports Grounds [2] states that:

> …where a seating deck has a vertical frequency of less than 6 Hz (…) a dynamic evaluation of the structure should be carried out, giving due consideration to the mass of spectators,

and also

> …at grounds staging pop concerts or other events involving rhythmic activity, design loads may be greater.

On the other hand, British Standard 6399 Part 1 [3] in its Section 9.2.1 states that:

> Dynamic loads will only be significant when any crowd movement (dancing, jumping, rhythmic stamping, aerobics, etc.) is synchronised. In practice, *this only occurs in conjunction with strong musical beat such as occurs at lively pop concerts or aerobics* [italicised by the authors of this paper]….

In addition to this, BS6399 advises that an assembly structure should be designed either to withstand anticipated human-induced dynamic loads (limited guidance as to the determination of this load is given in its Annex A) or to have a first natural frequency above 8.4 Hz so as to avoid resonance under the above loading.

The section in italics above has recently been shown to be misleading. An incident occurred at Liverpool Football Club in 2000 where significant motion was observed under action of a crowd that was not animated by music, and for a seating deck whose vertical natural frequency was found to be only 2 Hz. Many other stands in the UK have relatively low natural frequencies (3–4 Hz), either because they were constructed before guidance was available or because the guidance given in Annex A of BS6399 [3] is currently viewed by many designers as over-conservative and somewhat detached from reality. This is the main reason why the 1996 version of BS6399 was recently modified and should not be used in conjunction with crowd dynamic loading on grandstands.

A similar approach, presented in the Canadian code [4], leads to a smaller predicted response, but only because the severity of the anticipated jumping activity is reduced.

The jumping force time histories, presented in Annex A of BS6399 are based on the half sine pulse loading [1]. Unfortunately, the only structure for which any validation of this loading model was attempted was a simply supported concrete beam with a natural frequency around 18 Hz. This beam 'feels' extremely rigid for someone jumping at 2 Hz. Other experimental measurement of loads produced by people jumping [5–7] took place on rigid floors.

Bearing all this in mind, the research described in this paper is aimed at the apparent gap in the knowledge as to if/how the perception of the motion affects the jumping forces that caused that motion. It is also anticipated that the ability of a group of people to act in a coordinated manner is likely to improve for more flexible structures where significant motion is perceived. This was recently demonstrated for walking loads in the case of highly publicised excessive lateral sway motion of the Millennium Bridge in London. The Liverpool

FC case also demonstrates this to some extent, considering that significant perceptible motion and crowd synchronisation occurred without any musical beat.

In the UK, a national Joint Working Group under the auspices of the UK Institution of Structural Engineers [8] and two government departments (Office of the Deputy Prime Minister—ODPM and Department for Culture, Media and Sports—DCMS) was set up in 2000 to examine this whole problem. So far, Interim Guidance [8] has been issued to provide minimum natural frequencies for new and existing grandstand designs. Special treatment is necessary for any existing stadium lying below this threshold. The working group recognised that this frequency tuning approach is ‘coarse grained and indirect’ [8] and it therefore hopes to provide new guidance for calculation of the dynamic response of flexible structures to crowd loading. This is where the data presented in this paper may become useful. The paper follows initial work by Yao et al. [9,10] and presents a full set of experimental results from an extensive series of tests performed by a subject jumping at a range of frequencies on a platform having a range of natural frequencies.

2. Test rig

To understand the fundamental interaction between a single person (test subject) and a flexible structure, a test rig needed to be constructed, where the structure behaved essentially as a single-degree-of-freedom vibrating system in the vertical direction, with motions in other directions constrained to be as near to zero as possible.

2.1. Rig design

Fig. 1 shows the test rig designed for this purpose. A support structure carries vertical rails upon which a horizontal platform slides up and down on linear bearings with low friction. The stiffness required to provide a restoring force to the platform when it moves up or down is provided by a high tensile steel cantilever spring, with an adjustable prop support used to vary the natural frequency.

The platform is guarded on all sides so as to prevent any serious accident occurring if the subject were to fall whilst jumping. The platform mass for the initial configuration tested is 180 kg. The range of natural frequencies for which the support structure was designed is 1.5–6 Hz. A typical response time history for the platform displacement following an initial disturbance (Fig. 2) indicates clear single-degree-of-freedom behaviour. The influence of the platform being positioned well off the ground is believed to be small. The test subject rapidly became used to this and was instructed to keep his posture upright, with eyes level.

2.2. Rig instrumentation

The rig is instrumented as follows for dynamic response measurements. The platform vertical acceleration is measured using a Honeywell QA-700 accelerometer and the corresponding displacement using an LVDT (model RDP DCTH-15000C).

The force applied by the subject when jumping on the platform may be measured in one of two ways:

(i) The force in the vertical link at a point just below the platform (Fig. 1) is measured using an Entran force transducer (model ELHS-T4M-10 kN). This transducer will provide an *indirect* estimate of the jumping force exerted by the subject once the effect of the platform inertia force (i.e. platform mass times its measured acceleration) is removed. Any friction present would introduce errors into the measurement.

(ii) An AMTI OR6-6 force plate is embedded within the platform floor so as to provide a *direct* measurement of the vertical force produced by the subject jumping, as well as two orthogonal horizontal force components. In this case, the (much smaller) inertia force associated with the ‘active’ mass of the force plate (5 kg) needs to be subtracted from vertical force measured by the plate.

As far as the measurement of the force is concerned, the force plate was found to produce better quality time histories than the force link, particularly where the test subject was not in contact with the platform and a zero

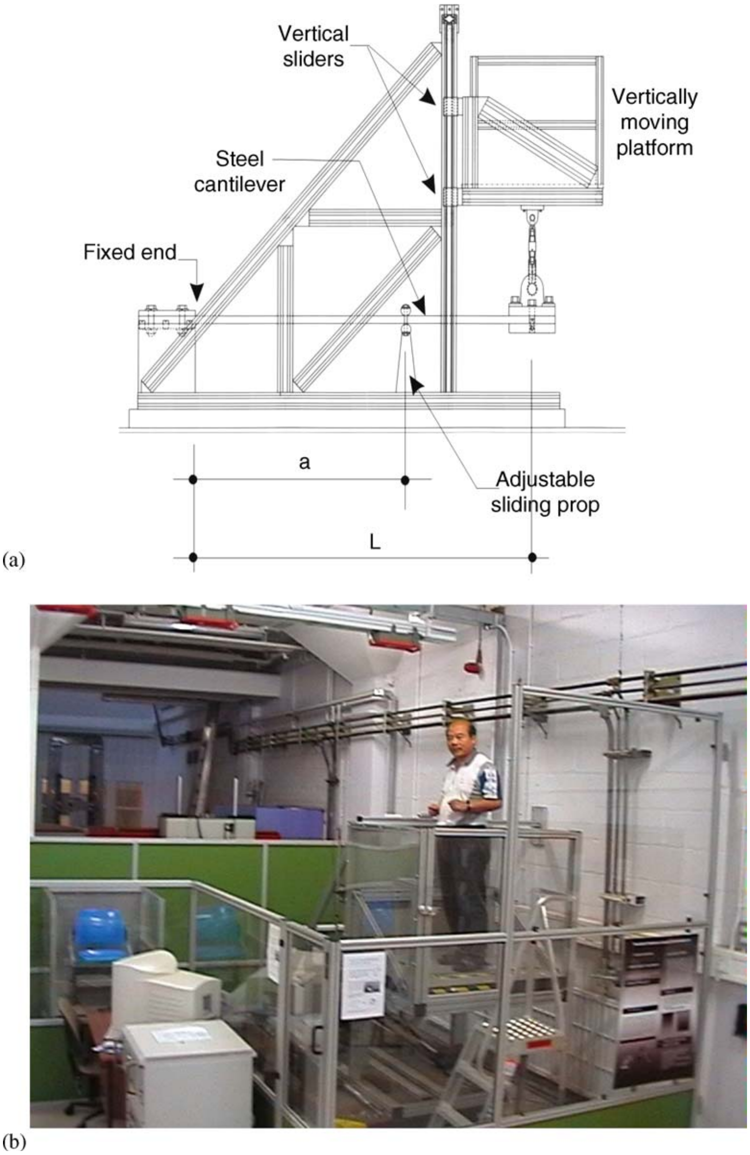


Fig. 1. Test rig: (a) rig structure, (b) rig in practical tests.

applied force was being measured. However, the consistency between the different approaches demonstrated that the instrumentation was functioning correctly.

Data were acquired using a PC-based multichannel National Instruments system with LabView software. It should be noted that all instrumentation was deliberately chosen to operate down to zero frequency (DC), so as to avoid any amplitude and phase errors at the very low frequencies which needed to be considered, especially given that signals had to be combined when removing the effect of the inertia forces.

3. Initial tests

3.1. Modal test

One critical issue is whether there are any modes of vibration with significant motion at the position of jumping that will make the rig behave differently to a single-degree-of-freedom system during jumping.

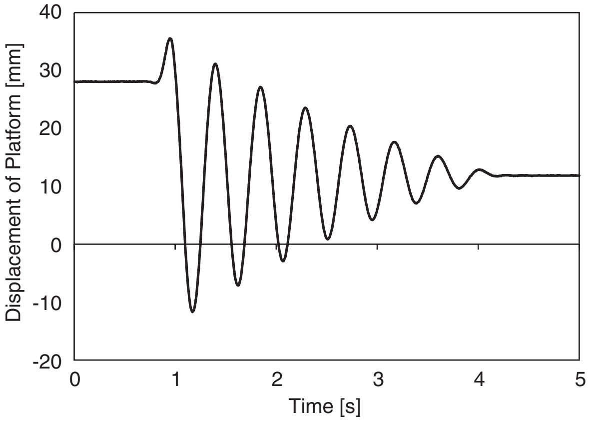


Fig. 2. Displacement of platform following an initial disturbance.

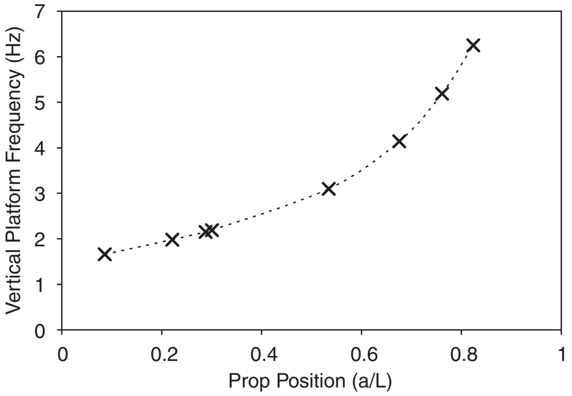


Fig. 3. Variation of vertical frequency with normalised prop position.

Therefore a modal test was performed, both by exciting the support frame with a modal hammer, and by exciting the platform in the vertical direction using an APS 113 electrodynamics shaker.

Some modes involving lateral motion of the support frame or platform were found above 15 Hz but these are unlikely to be excited strongly by the small horizontal components of force that will be generated by any subject on the platform. As expected from preliminary calculations with a finite element model, the first mode with significant vertical motion at the jumping position was above 90 Hz. It can be shown theoretically that this is not likely to be a problem for the rather limited range of excitation frequencies concerned. Therefore, these checks demonstrated that the platform is essentially behaving as a 180 kg single-degree-of-freedom system under human-induced dynamic loading.

The modal test using the shaker was performed for a series of prop positions to allow the variation of natural frequency with normalised non-dimensional prop position a/L (Fig. 1) to be determined and fitted. Fig. 3 shows this variation, which is clearly smooth and demonstrates that the desired range of natural frequencies from 1.5 to 6 Hz may be achieved by adjustment of the prop. Further shortening of the cantilever (Fig. 1) leads to a maximum natural frequency of 16 Hz of the test rig that was examined in this work.

Of particular concern was the linearity of the behaviour of the desired vertical mode of the platform mass moving as a rigid body upon the cantilever ‘spring’. A swept sine excitation was applied across a narrow frequency band around this mode at different excitation levels and some degree of nonlinearity was noted. The behaviour of the measured frequency response function (FRF) between platform acceleration and applied force corresponded to what might be expected were some friction to be present. Whilst the friction levels experienced when attempting to initiate motion (even by hand let alone by jumping action) seem to be very small, there was a noticeable effect on the FRFs produced at the comparatively low force levels introduced in the modal test. For example, the natural frequency for one spring configuration changed from 3.11 to 3.17 Hz when the force level was halved, indicating an apparent 'stiffening' due to increased friction resistance.

Given that some relative motion is present in the vertical sliders, in the bearings at each end of the vertical link and in the clamps at the beam root and prop positions, it is considered inevitable that some friction will be present. At this stage, it is considered that the level of friction is not a cause for concern. Free decays of the platform motion following a disturbance were very smooth (Fig. 2) and only a single frequency was apparent.

3.2. Damping

The amount of damping also depended on the platform displacement as shown in Fig. 4, where damping values were estimated on a cycle-by-cycle basis during the decay for the rig set at a natural frequency of 2.5 Hz. Two tests are shown, corresponding to different initial conditions for the free decay. The high level of damping at very low amplitudes (small initial disturbance) shows where friction dominates the damping. However, at higher amplitudes of oscillation (large initial disturbance), the damping tends to a constant value, where the basic structural damping dominates. The latter region of behaviour is more relevant to the tests in this paper, particularly where the response is large and therefore of most importance. Simulations to validate any human–structure interaction model can include both damping terms.

A curve fit applied to Fig. 4 using a model that is equivalent to viscous damping due to friction, together with a linear damping term, yielded an equivalent damping value of 1.5% critical and a friction of approximately 25 N, which is small compared to the magnitude of the applied force of the order of 1 kN.

3.3. Calibration checks

Initial static calibrations of the force plate, force gauge in the link, LVDT and the 'turnover test' of the force-balance accelerometer were carried out to check the manufacturer's calibration data and acquisition set-up. A comparison of force values determined from the force plate and the force gauge in the link is shown in Fig. 5 for a sequence where the subject first stands, then bounces and finally jumps. It may be seen that there is good agreement between the two approaches, though the force plate provides a more accurate measurement in the region where the jumper loses contact with the platform and the force value should be zero.

4. Procedure for jumping tests

All tests presented are from a single male test subject (Dr Yao) whose mass was 75 kg. Only portions of the measured data, corresponding to as steady-state response as possible, were used in the analysis described below.

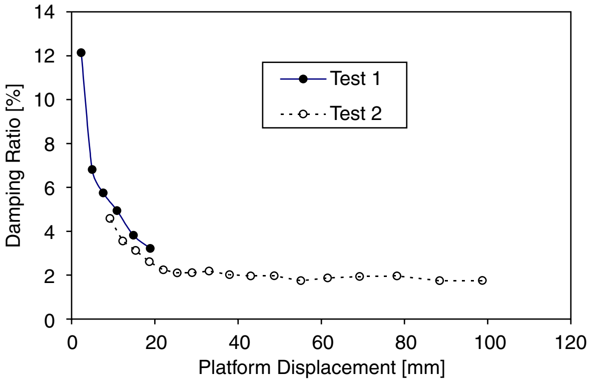


Fig. 4. Variation of equivalent viscous damping ratio with peak displacement of a platform having 2.5 Hz natural frequency.

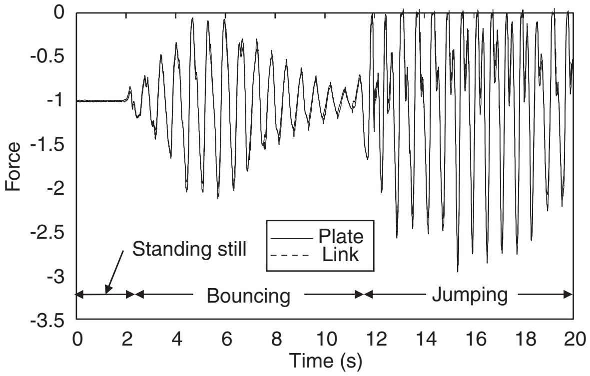


Fig. 5. Comparison of forces measured from both force plate and force gauge in link.

4.1. Test parameters

All force measurements shown are normalised to the subject's weight, which was 736 N. As the moving mass of the platform was 180 kg, the subject/platform mass ratio was approximately 0.41. This value is within an acceptable range for real grandstand structures when filled to their full capacity (0.25–0.75). Also, all platform acceleration results are normalised to the gravitational constant ($g = 9.81\,\mathrm{m/s^2}$), and all platform displacement results are normalised to the static sag caused by the weight of the test subject (i.e. not including the sag of the platform under its own weight). Therefore, the normalised displacement is effectively a dynamic magnification based on the static effect of the subject.

The following values of the vertical natural frequency of the empty platform were used: 2, 2.5, 3, 3.5, 4, 5, 6 and16 Hz, the last value representing a 'pseudo-rigid' behaviour of the test rig.

4.2. Targeted and achieved jumping frequencies

The test subject aimed to jump at a range of frequencies between 1 and 3.5 Hz, with the aid of a metronome. However, one of the important outcomes of these tests is that these so-called *'targeted'* jumping frequencies (f_{jT}) were not always met. Therefore, the term *'achieved'* jumping frequency (f_{jA}) was adopted and it corresponds to the frequency of the fundamental harmonic of the force time history. This frequency was identified as the frequency of the first peak in the measured force spectrum.

In addition to tests where a particular jumping frequency was sought, other tests were performed where the subject was asked to jump 'freely' at a frequency that felt most comfortable to him for the platform configuration under test.

For each test, the subject sought to maintain a steady jumping motion for about 20 s. Once the test was complete, a portion of the time history was selected where the force and responses exhibited near steady-state behaviour. These data were then processed to yield maximum and minimum values in the time domain, jumping contact ratio (see Section 4.3 below) and values of the first, second and third harmonic peaks in the Fourier spectra.

Some sample time history results are shown in Fig. 6 for jumping at 2 Hz on a 4 Hz platform (i.e. platform configured to have a 4 Hz natural frequency), and in Fig. 7 for attempting to jump at 2 Hz on a 2 Hz platform, but only achieving 1.8 Hz. Accelerations of 1.5–2.2 g were achieved in these examples.

4.3. Contact ratio

When a subject jumps, the contact ratio (α) is defined as the proportion of the jumping cycle in which the subject is in contact with the platform. Because the average force per jumping cycle must be equal to the subject's weight, a low contact ratio corresponds to jumping where the force peak is high whereas a high contact ratio corresponds to a much lower peak force. In Annex A of BS6399 [3], force histories are defined for several contact ratios, namely 1/4 (high impact jumping), 1/3 (normal jumping), 1/2 (rhythmic exercise)

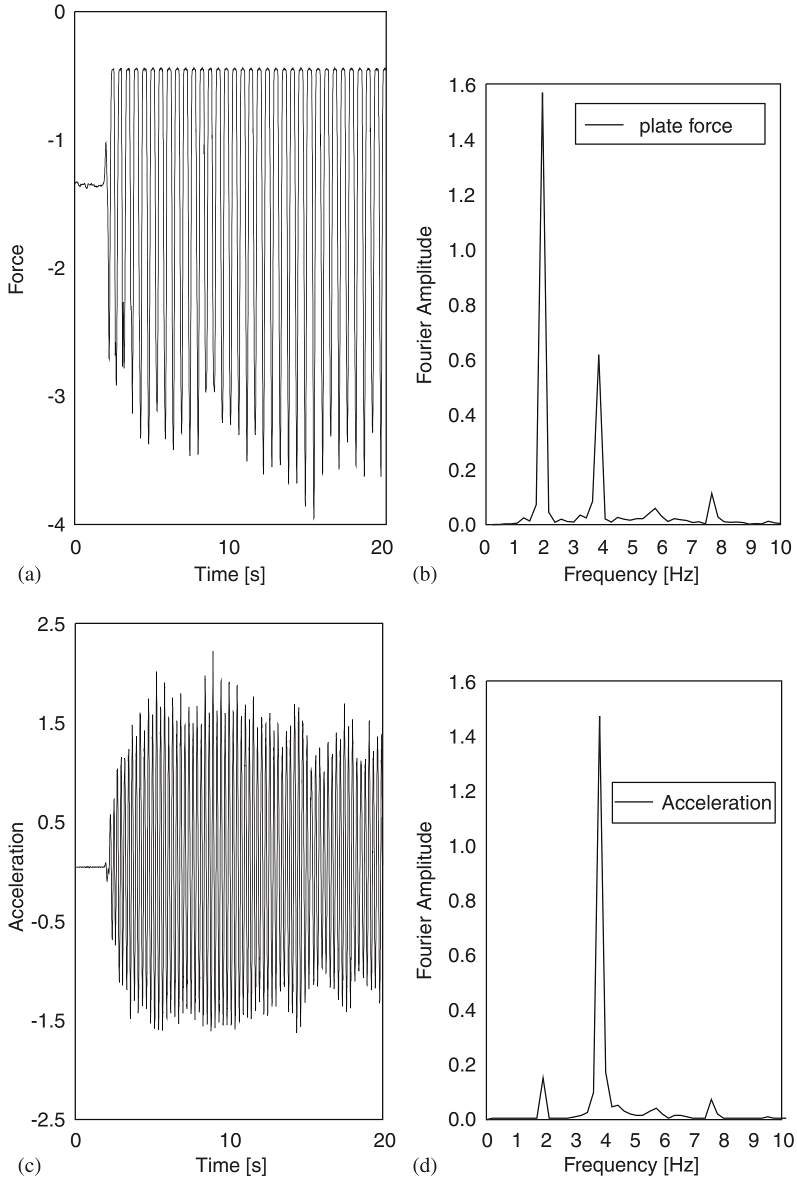


Fig. 6. Time histories and spectra for jumping at 2 Hz on a 4 Hz platform: (a) force in time domain, (b) force in frequency domain, (c) acceleration in time domain, (d) acceleration in frequency domain.

and 2/3 (low impact jumping). As previously mentioned, because no formal tests have ever been reported for jumping on flexible structures, these definitions are believed to correspond to jumping on rigid surfaces.

In the tests reported in this paper, the contact ratio was estimated from the force time history by determining the proportion of time for which the measured force lay below a threshold (typically 5–10% of the peak force).

5. Results from jumping tests

5.1. Jumping frequency

The ability of the subject to achieve the targeted jumping frequencies is shown in Fig. 8 where the ratio f_{jA}/f_{jT} is plotted against the achieved jumping frequency normalised by the platform natural frequency (f_{jA}/f_P).

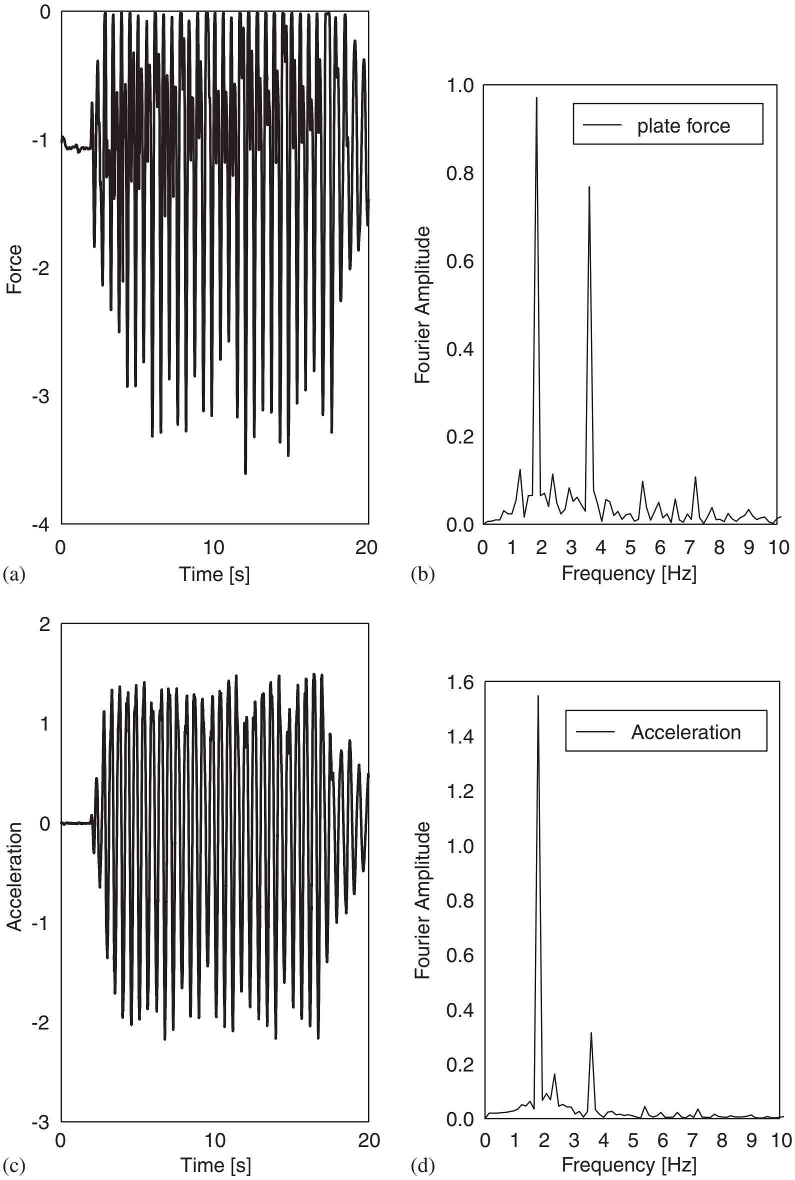


Fig. 7. Time histories and spectra for attempting to jump at 2 Hz on a 2 Hz platform (frequency achieved was only 1.8 Hz): (a) force in time domain, (b) force in frequency domain, (c) acceleration in time domain, (d) acceleration in frequency domain.

In interpreting this figure, it should be noted that the frequency resolution of the frequency spectra is of the order 0.25–0.125 Hz because of the limited amount of steady-state data available (typically 4–8 s).

What is apparent from Fig. 8 is that the test subject was unable to jump at frequencies (2, 2.5, 3 and 3.5 Hz) that would excite closely the platform's resonance when it moves significantly. The flexibility of the platform meant that it was not possible to 'take off' and maintain jumping at these frequencies. Indeed, in some cases the achieved frequency was approximately 20% below or 10% above the targeted frequency. The most likely explanation for the inability to jump at the natural frequency, and so achieve the peak resonant response, is that the motion is so significant that, if it were any larger, the subject acting as a projectile in the airborne phase would not land in time to maintain jumping motion at the defined frequency. Typically, at such high motion levels the test subject was 'forced' to abandon his prescribed jumping frequency, and 'switch' to the predominant frequency of the motion of the combined human–structure dynamic system, which was not the

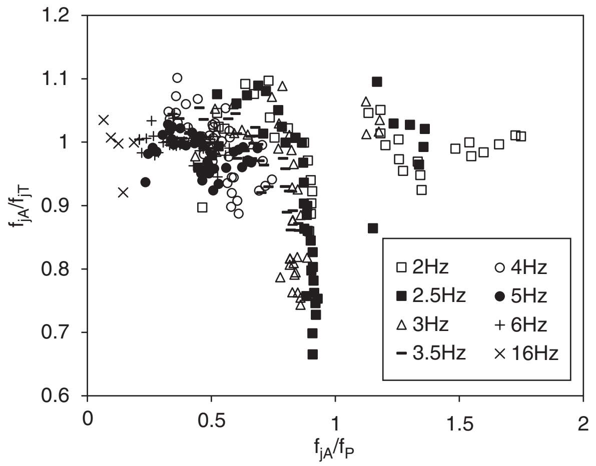


Fig. 8. Variation of the ratio of achieved and targeted jumping frequencies (f_{jA}/f_{jT}) with f_{jA}/f_P for different natural frequencies.

same as the frequency f_P of the empty test rig. Further research is underway in assessing the motion of the jumper relative to the platform using a video system. Once this work is complete, an analysis of the projectile explanation should be possible.

The reason for this interesting behaviour is not yet fully understood but it is supported by early theoretical studies on a method for predicting human–structure interaction which description is beyond the scope of this paper. It suffices to say here that it is likely that a higher platform mass (and stiffness, to maintain the natural frequency unchanged), as well as higher damping, would reduce the platform motion and this effect, too.

5.2. Contact ratio

The contact ratios for the entire set of tests at all the natural and jumping frequencies are presented in Fig. 9 as a function of the ratio f_{jA}/f_P. What is notable is that a wide range of contact ratios was determined. However, as previously mentioned, none were below 0.5, even for the platform with a natural frequency of 16 Hz (‘pseudo-rigid’). This is an interesting observation compared with what is suggested in BS6399 [3], which was apparently based on results of jumping tests on rigid surfaces but without hard evidence that such low contact ratios are achievable [11,12].

It is apparent that there is an increase of contact ratio in the regions where jumping occurred near to the natural frequency and at around half the platform natural frequency. This would correspond to ‘low impact jumping’, to use BS6399 terminology mentioned before. The whole phenomenon is, almost certainly, a consequence of the greater motion of the platform when one of the harmonic components of the jumping force excited a near resonant response of the platform. As the response increased, the motion tended towards more of a low- or no-impact ‘bouncing’ type. Away from resonance regions, the contact ratio was in the range 0.5–0.7.

It is yet to be seen whether test subjects can adapt their jumping motion to a specific pre-determined contact ratio for a flexible structure. Early results so far indicate that it is likely to be very difficult to achieve, and that a test subject feeling vibration ‘automatically’ selects only the contact ratio he/she is most comfortable with when jumping (i.e. the one which requires least effort).

5.3. Jumping force

The variation of the first and second harmonics of the force is presented in Fig. 10 as a function of f_{jA}/f_P. These results are extracted from the force spectral peaks at the achieved jumping frequency and twice its value.

As seen in Fig. 10a, the bulk of the first harmonic DLF results lie between 1.2 and 1.7. These values correlate well with force values tabulated in the BS6399 [3] half sine force model for contact ratios in the range

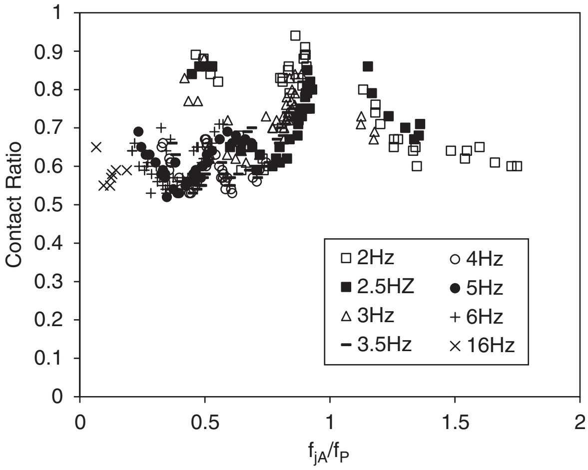


Fig. 9. Variation of contact ratio with f_{jA}/f_P for different natural frequencies.

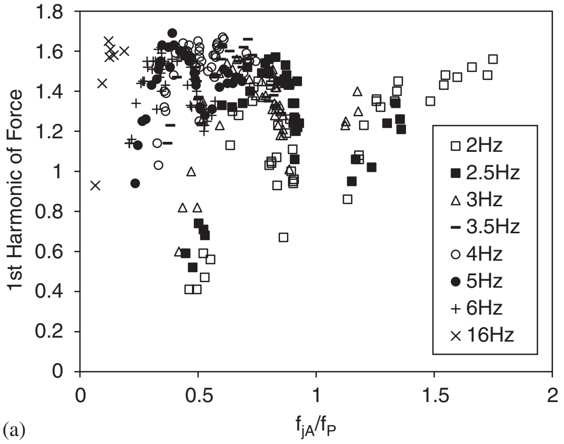
0.7–0.4. However, for jumping at half the platform natural frequency and close to the natural frequency itself, the first harmonic DLF falls to values between 40% and 100% of the weight. This corresponds to values tabulated for contact ratios between 1 and 0.8, which is more like ‘bouncing’ than jumping. This result indicates that when the motion is considerable, the test subject is somehow ‘forced’ out from jumping into bouncing. The second harmonic DLF in Fig. 10b varies from almost 0 to maximum of 1, corresponding to values tabulated for contact ratios between 0.8 and 0.4. Note that in the BS6399 [3] table of values, the second harmonic DLFs are particularly sensitive to contact ratio. Hence, considering this and greater randomness of the second harmonic DLFs [13] it is not surprising that there is such a wide variation of values in Fig. 10.

The maximum values of force from the time domain results will depend upon the combination of several harmonics where phase may differ. The measured force peaks are shown in Fig. 11 and it may be seen that the maximum force varies between 1.8 and 4 times the subject weight. This is commensurate with the change in contact ratio. There is some noticeable reduction in overall force level around f_{jA}/f_P ratio values of 1/2 and 1. This reduction might be thought to be analogous to the phenomenon of force ‘drop-out’ experienced when using electrodynamic shakers for modal testing, a feature that occurs due to the inertia of the shaker mass itself, if the subject can be thought of as providing an inertial force excitation to the platform [14].

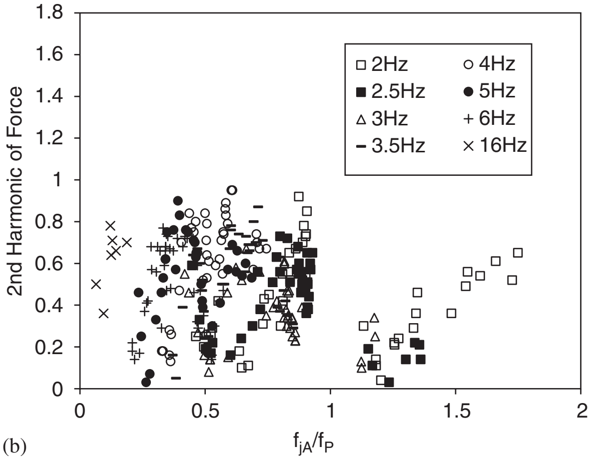
What is clear from the measured force data is that the variation of contact ratio found means in turn that the DLFs vary considerably with the amount of motion, i.e. with the f_{jA}/f_P ratio and the platform natural frequency f_P itself. Therefore, in the case when a perceptible vertical motion is expected, it is not advisable to simply adopt a contact ratio for a particular type of jumping (e.g. normal jumping), as advised in Annex A of BS6399 [3]. In this case, it is also prudent to consider the amount of structural motion expected and adjust the human-induced excitation levels as appropriate.

5.4. *Acceleration*

If the acceleration response of the test structure is assumed to be periodic, amplitudes of its first two harmonics are presented in Fig. 12 for a range of tests. The first harmonic results are increasing as the frequency ratio f_{jA}/f_P approaches 1 (i.e. jumping near to the natural frequency). The results look rather like a non-resonant part of a classical frequency response curve. In the resonant region of this curve the apparent peak could not be reached because of the difficulty of achieving the target frequency under large amplitude motion. The peak acceleration of the first harmonic is about 1.9 g, which is an extremely large value in the civil engineering context when designing lively structures dynamically occupied and excited by humans, such as grandstands, footbridges or staircases. It should be noted that this value of acceleration is considerably higher than what has been encountered in real civil engineering structures where accelerations of more than 35–50% g are rare and would almost certainly cause disruption of the normal service of the structure.



(a)



(b)

Fig. 10. Variation of (a) first and (b) second harmonics of measured force with f_{jA}/f_P for different natural frequencies.

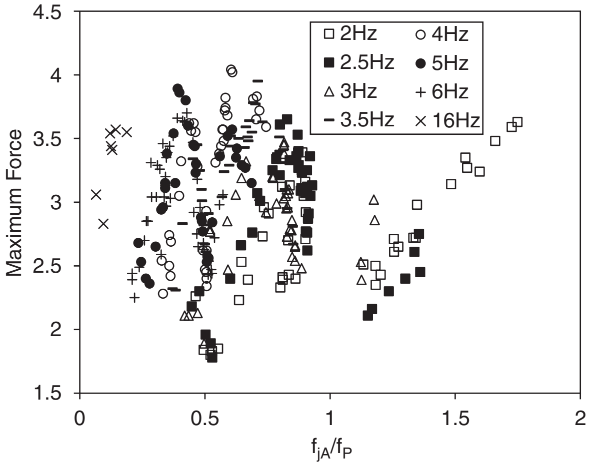


Fig. 11. Variation of maximum force with f_{jA}/f_P for different natural frequencies.

It is interesting that the results for the different platform natural frequencies f_P almost overlay. This is not unreasonable given that most of the first harmonic force values are fairly similar when f_{jA}/f_P is less than 0.9 (Fig. 10a). The appropriate expression for the acceleration response of a single-degree-of-freedom

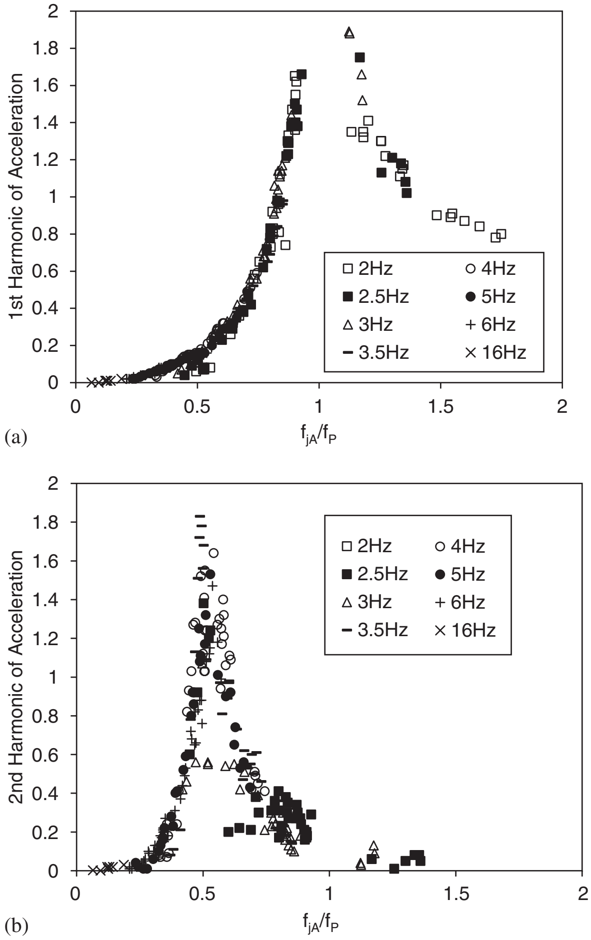


Fig. 12. Variation of (a) first and (b) second harmonics of acceleration with f_{jA}/f_P for different natural frequencies.

system, expressed as a function of a non-dimensional ratio of the excitation and natural frequencies, shows that results would then collapse onto one curve for constant damping values [15], as shown in Fig. 12a.

It may be seen from Fig. 12 that the second harmonic of acceleration peaks when the jumping frequency is around half of the natural frequency. This is, again, with a 'resonance-like' appearance and with a similar acceleration of 1.8 g. The acceleration harmonic values are somewhat larger for the higher natural frequencies. This is so, perhaps, because the jumping frequencies are higher (2 Hz or more) and easier to achieve than the values of 1–1.5 jumps/s required to provide excitation at half the lower natural frequencies. Indeed, at very low jumping frequencies (say, less than 1.5 Hz), the nature of the test subject's jumping motion changed because both the heel and toe made contact with the surface but at different times. Therefore, there was effectively a double peak in each cycle of jumping which emphasised the amplitude of the second harmonic.

What is interesting is that the peak acceleration values are similar in the two peak regions whereas it might be expected that the response would be larger when $f_{jA}/f_P = 1$. As before, this is most likely because it has not been possible to excite the perceptibly moving structure at exactly the natural frequency where much larger responses occur.

Finally, overall acceleration results are presented in Fig. 13, with each maximum value extracted from the time history. This figure shows peaks of the f_{jA}/f_P ratio around 1/2 and 1, and possibly even at 1/3. The maximum peak acceleration achieved in the tests is around 2.2 g.

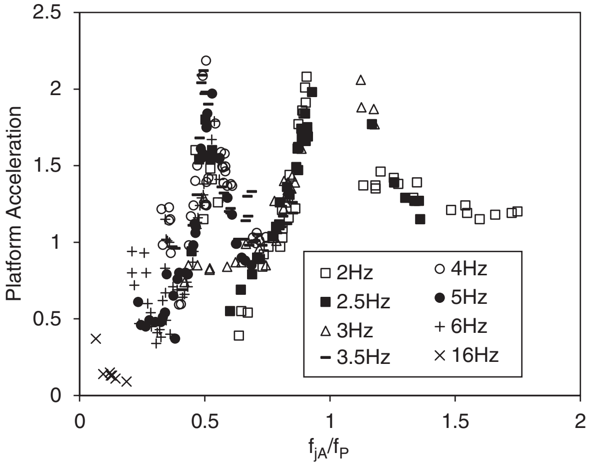


Fig. 13. Variation of maximum acceleration with f_{jA}/f_P for different natural frequencies.

5.5. Displacement

As for the acceleration results, the displacement of the test structure is also assumed to be periodic, and the amplitudes of its first two harmonics are presented in Fig. 14.

It can be seen that the shapes of the curves for the variation of normalised displacement with the f_{jA}/f_P ratio are very similar in shape to the acceleration results shown earlier. Peaks are also around values of the f_{jA}/f_P ratio of 1/2 and 1, and possibly even at 1/3 where the third harmonic may excite higher values of the fundamental natural frequency of the platform. The maximum displacement achieved is about ±135 mm for the 2 Hz platform setting (presented in Fig. 15). It should be noted that not only are the perceived acceleration values important for the crowd experiencing them first-hand, but also the level of displacement of any structure in the line of sight of the spectator may also be alarming. The displacements are also important as they are directly linked to the structural strength.

6. Concluding remarks

The most significant finding of the experimental study presented in this paper has been that it is not possible to jump at (or very near) to the natural frequency of a structure that moves significantly. It is considered that as the response increases with the onset of resonance, the time spent by the subject in the projectile motion during the airborne phase of the jumping motion provides a limiting condition. For the case considered, the limiting acceleration is found to be around 2 g.

It is also apparent that the contact ratio varies with natural frequency, typically in the range 0.5–0.7, but increasing in the regions of near-resonant response to 0.75–0.95. Contact ratios were not achieved below a value of 1/2, in contrast with the values of 1/4 and 1/3 quoted in BS6399. As the response amplitude increases, it appears that the motion of the subject tends towards near-bouncing behaviour.

The corresponding force harmonic values show a range of values reasonably consistent with the BS6399 harmonics for similar contact ratios, indicating that the shape of each jumping force pulse is not dissimilar to the half sine wave assumed in BS6399. However, the force values drop in the regions in which resonant or near resonant excitation occur.

Finally, the behaviour of the non-dimensional acceleration harmonics for different platform natural frequencies was very similar in non-resonant regions where the structural motion was less significant. This indicates relative independence of the structural response from the natural frequency of the empty structure when the motion is less perceptible by the jumper during jumping.

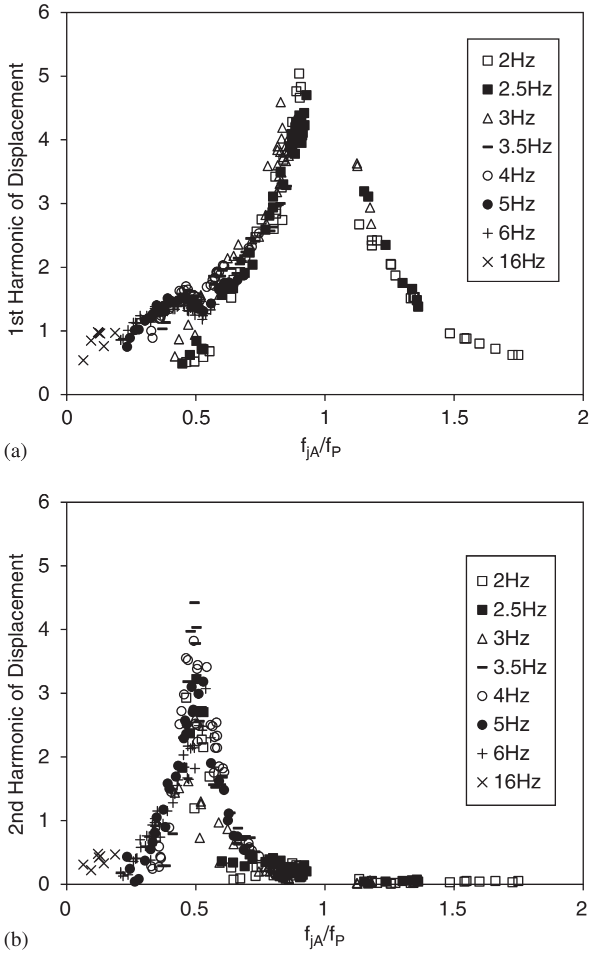


Fig. 14. Variation of first and second harmonics of displacement with f_{jA}/f_P for different natural frequencies.

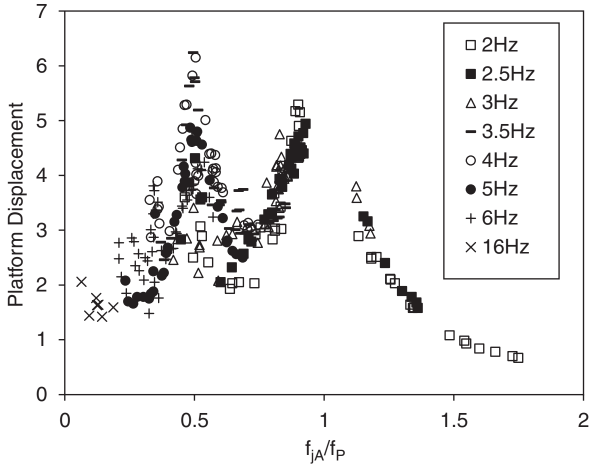


Fig. 15. Variation of maximum displacement with f_{jA}/f_P for different natural frequencies.

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